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THERMAL INFLUENCE ON THE STRESS-STRAIN STATE OF ELASTIC THICK-WALLED PIPE

Annotation. Solutions for the stress-strain state of a thick-walled pipe subjected to internal and external pressures were obtained, accounting for temperature effects under plane strain conditions. A comparative graphical analysis was performed to evaluate the results with and without thermal influence.

Keywords: Thick-walled pipe, strain, volumetric expansion, radial displacement, stress, coefficient of thermal expansion.

ELASTIK QALIN DEVORLI QUVURNING KUCHLANGAN-DEFORMATSIYALANGAN HOLATIGA TEMPERATURA TA’SIRI

Annotatsiya: Ichki va tashqi tomonlaridan ta’sir etuvchi bosim ostidagi qalin devorli quvurning kuchlangan-deformatsiyalangan holati, temperatura ta’siri hisobga olingan, tekis deformatsiya holatida yechimlari olindi. Olingan natijalar temperatura ta’siridan holi va e’tiborga olingandagi grafik ko‘rinishidagi natijalar taqqoslandi.

Kalit so‘zlar: Qalin devorli quvur, deformatsiya, hajmiy kengayish, radial ko‘chish, kuchlanish, temperaturaviy kengayish koeffisienti.

ТЕМПЕРАТУРНОЕ ВЛИЯНИЕ НА НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ УПРУГОЙ ТОЛСТОСТЕННОЙ ТРУБЫ

Аннотация. Получены решения задачи о напряженно-деформированном состоянии толстостенной трубы, находящейся под воздействием внутреннего и внешнего давления, с учетом температурного влияния в условиях плоской деформации. Проведен сравнительный анализ результатов в графическом виде для случаев с учетом и без учета температурного воздействия.

Ключевые слова: Толстостенная труба, деформация, объемное расширение, радиальное перемещение, напряжение, коэффициент температурного расширения.

Introduction. The problem of pipe analysis dates back to the early 19th century and can be found in the pioneering works of the Russian academician A.I. Gadolin, the French mathematician G. Lamé, and the German scientist T. von Kármán. Specifically, A.I. Gadolin performed strength and stiffness calculations for firearm barrels. The research conducted by Kármán was primarily devoted to studying the stress-strain state of thin-walled pipes. Given the wide range of applications for piping systems and the significant impact of thermal loads during their operation, this article investigates the stress-strain state of thick-walled pipes while accounting for temperature effects. In this study, a practical problem is solved, and relevant conclusions are drawn based on the results.

Problem Statement. Due to the axisymmetric loading acting on the elastic pipe, its deformation is symmetric with respect to the longitudinal z-axis. Considering that all characteristics of the stress-strain state within the pipe's cross-section are independent of the z-coordinate, the plane strain state is analyzed in a cylindrical coordinate system (Fig. 1). Consequently, only the radial displacement u is non-zero, while all other displacement components are equal to zero (vanish).

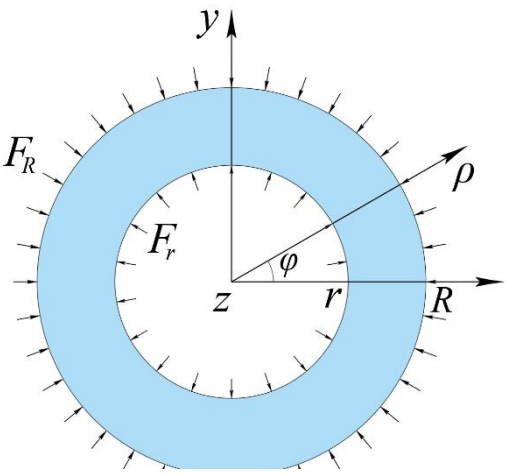


Figure 1. Elastic pipe under loading

The expression of strains in terms of displacements in cylindrical coordinates for the plane strain state:

$$\varepsilon_\rho=\frac{du}{d\rho};\varepsilon_\phi=\frac{u}{\rho} \qquad (1)$$

For the plane strain state, the relationship between normal stress and strain, accounting for thermal effects, is determined by Hooke's law as follows:

$$\sigma_\rho=2G\varepsilon_\rho+\lambda(\varepsilon_\rho+\varepsilon_\phi)-3K\alpha T;$$
$$\sigma_\phi=2G\varepsilon_\phi+\lambda(\varepsilon_\rho+\varepsilon_\phi)-3K\alpha T;$$
$$\sigma_z=\lambda(\varepsilon_\rho+\varepsilon_\phi)-3K\alpha T; \qquad (2)$$

№	Material properties	Formulas
1	Shear modulus	$G=\frac{E}{2(1+\nu)};$
2	Lame constants	$\lambda=\frac{\nu}{(1-2\nu)}\frac{E}{(1+\nu)};$
3	Volumetric strain	$\theta=\varepsilon_\rho+\varepsilon_\phi+\varepsilon_z;$
4	Bulk modulus	$K=\frac{E}{3(1-2\nu)};$
5	Young's modulus	$E=E(T)$
6	Poisson's ratio	$\nu=\nu(T)$
7	Coefficient of thermal	$\alpha=\alpha(T)$

	expansion	

Table 1. Material properties and their expressions.

The equilibrium equation in cylindrical coordinates for the plane strain state is given as follows:

$$\frac{d\sigma_\rho}{d\rho}+\frac{\sigma_\rho-\sigma_\Phi}{\rho}=0 \qquad (3)$$

The boundary conditions for the problem under internal and external pressures are defined as follows:

$$\sigma_\rho\big|_{\rho=r}=-F_r$$
$$\sigma_\rho\big|_{\rho=R}=-F_R \qquad (4)$$

Solution of the problem. By substituting the expressions for strains and stresses into the equilibrium equation (3), the governing equation is derived:

$$\frac{d^2u}{d\rho^2}+\frac{du}{\rho d\rho}-\frac{u}{\rho^2}=\frac{1+\nu}{1-\nu}\alpha\frac{dT}{d\rho} \qquad (5)$$

According to $\left(\frac{d^2u}{d\rho^2}+\frac{du}{\rho d\rho}-\frac{u}{\rho^2}\right)=\frac{d}{d\rho}\left(\frac{du}{d\rho}+\frac{u}{\rho}\right)$, we can rewrite the equation (5) as follows:

$$\frac{d}{d\rho}\left(\frac{du}{d\rho}+\frac{u}{\rho}\right)=\frac{1+\nu}{1-\nu}\alpha\frac{dT}{d\rho} \qquad (6)$$

After integrating both sides of equation (6), we obtain the following:

$$\frac{du}{d\rho}+\frac{u}{\rho}=\frac{1+\nu}{1-\nu}\alpha T+C_1 \qquad (7)$$

By performing the substitution $\frac{du}{d\rho}+\frac{u}{\rho}=\frac{1}{\rho}\frac{d}{d\rho}(u\rho)$ and integrating expression (7) once more, we arrive at the following result:

$$u=\frac{1+\nu}{1-\nu}\alpha\frac{J}{\rho}+C_1\rho+\frac{C_2}{\rho} \qquad (8)$$

Where J is the temperature functional

$$J=\int_r^\rho T\rho d\rho \qquad (9)$$

The strains are determined as follows:

$$\varepsilon_\Phi=\frac{u}{\rho}=\frac{1+\nu}{1-\nu}\alpha\frac{J}{\rho^2}+C_1+\frac{C_2}{\rho^2} \qquad (10)$$

$$\varepsilon_\rho=\frac{du}{d\rho}=\frac{1+\nu}{1-\nu}\alpha T-\frac{1+\nu}{1-\nu}\alpha\frac{J}{\rho^2}+C_1-\frac{C_2}{\rho^2} \qquad (11)$$

By substituting expressions (10) and (11) into the stress expressions (2), the normal stresses varying along the coordinate ρ are determined:

$$\sigma_\rho=\frac{E}{1-\nu}\left[\frac{1-\nu}{1+\nu}\left(\frac{C_1}{1-2\nu}-\frac{C_2}{\rho^2}\right)-\alpha\frac{J}{\rho^2}\right] \quad (12)$$

$$\sigma_\phi=\frac{E}{1-\nu}\left[\frac{1-\nu}{1+\nu}\left(\frac{C_1}{1-2\nu}+\frac{C_2}{\rho^2}\right)+\alpha\frac{J}{\rho^2}-\alpha T\right] \quad (13)$$

$$\sigma_z=\frac{E}{1-\nu}\left[\frac{1-\nu}{1+\nu}\frac{2C_1\nu}{1-2\nu}-\alpha T\right] \quad (14)$$

The integration constants C_1 and C_2 are determined from the boundary conditions (4).

$$-F_r=\frac{E}{1-\nu}\left[\left(\frac{C_1}{1-2\nu}-\frac{C_2}{r^2}\right)\right] \quad (15)$$

$$-F_R=\frac{E}{1-\nu}\left[\frac{1-\nu}{1+\nu}\left(\frac{C_1}{1-2\nu}-\frac{C_2}{R^2}\right)-\alpha\frac{J_R}{R^2}\right] \quad (16)$$

Where

$$J_R=\int_r^R T\rho d\rho \quad (17)$$

Solving the system of equations 15 and 16, we obtain the following results:

$$C_1=\frac{1+\nu}{1-\nu}\frac{1-2\nu}{R^2-r^2}\left[\frac{1-\nu}{E}(F_r r^2-F_R R^2)+\alpha J_R\right] \quad (18)$$

$$C_2=\frac{1+\nu}{1-\nu}\frac{r^2}{R^2-r^2}\left[\frac{1-\nu}{E}(F_r R^2-F_R R^2)+\alpha J_R\right] \quad (19)$$

The coefficient of linear thermal expansion is determined from experimental data using the following formula:

$$\alpha=\frac{l_0-l_1}{l_0\Delta T} \quad (20)$$

Where l_0 is the length corresponding to temperature T_0, l_1 is the length corresponding to temperature T_1, and ΔT is the temperature increment.

Using the determined values of the integration constants, the radial displacement, strains, and stresses for the plane strain problem are calculated by means of the following equations.

$$u=\frac{1+\nu}{1-\nu}\left\{\alpha\frac{J}{\rho}+\frac{\alpha J_R\rho}{R^2-r^2}\left(1-2\nu+\frac{r^2}{\rho^2}\right)+\frac{(1-\nu)\rho}{E(R^2-r^2)}\left[(1-2\nu)(F_r r^2-F_R R^2)+\frac{r^2R^2}{\rho^2}(F_r-F_R)\right]\right\}$$

$$\sigma_\rho=\frac{E}{1-\nu}\left\{-\alpha\frac{J}{\rho^2}+\frac{1}{R^2-r^2}\left[\frac{1-\nu}{E}\left((F_r r^2-F_R R^2)-\frac{r^2R^2}{\rho^2}(F_r-F_R)+\alpha J_R\left(1-\frac{r^2}{\rho^2}\right)\right)\right]\right\}$$

$$\sigma_\phi=\frac{E}{1-\nu}\left\{-\alpha T+\alpha\frac{J}{\rho^2}+\frac{1}{R^2-r^2}\left[\frac{1-\nu}{E}\left((F_r r^2-F_R R^2)+\frac{r^2R^2}{\rho^2}(F_r-F_R)+\alpha J_R\left(1+\frac{r^2}{\rho^2}\right)\right)\right]\right\}$$

$$\sigma_z=\frac{E}{1-\nu}\left\{-\alpha T+\frac{2\nu}{R^2-r^2}\left[\frac{1-\nu}{E}\left((F_r r^2-F_R R^2)+\alpha J_R\right)\right]\right\}$$

Practical Problem. A thick-walled pipe with an inner radius r=45 mm and an outer radius R=75 mm undergoes axisymmetric elastic deformation under the influence of internal pressure F_r=20MPa and external pressure F_R=4 MPa. The pipe's material properties are defined by its Young's modulus E=$(200+0{,}1t)\cdot 10^3 MPa$, Poisson's ratio ν=0.3, and coefficient of thermal expansion α=$(12+0{,}01t)\cdot 10^{-6}\text{C}^{-1}$. A temperature field is described by the function $T(\rho)$=$60+80\cdot e^{-0.08(\rho-45)}$. For the purpose of this analysis, displacements and stresses are evaluated by neglecting the effects of temperature variation. The radial coordinate is considered to change according to the law ρ=$r+k\Delta\rho$, $\Delta\rho$=3mm, where k=0,…,10. Numerical results and their graphical representations were obtained for 10 radial increments within the region between the inner and outer radii.

№	ρ, mm	u, mm	σ_ρ, MPa	σ_ϕ, MPa	σ_z, MPa
$t\neq 0$	60	4.6487	−9.0626	20.417	2.9994
t=0	60	5.484	−9.0625	19.0625	3

Table 2. The Results of the problem for case k=5.

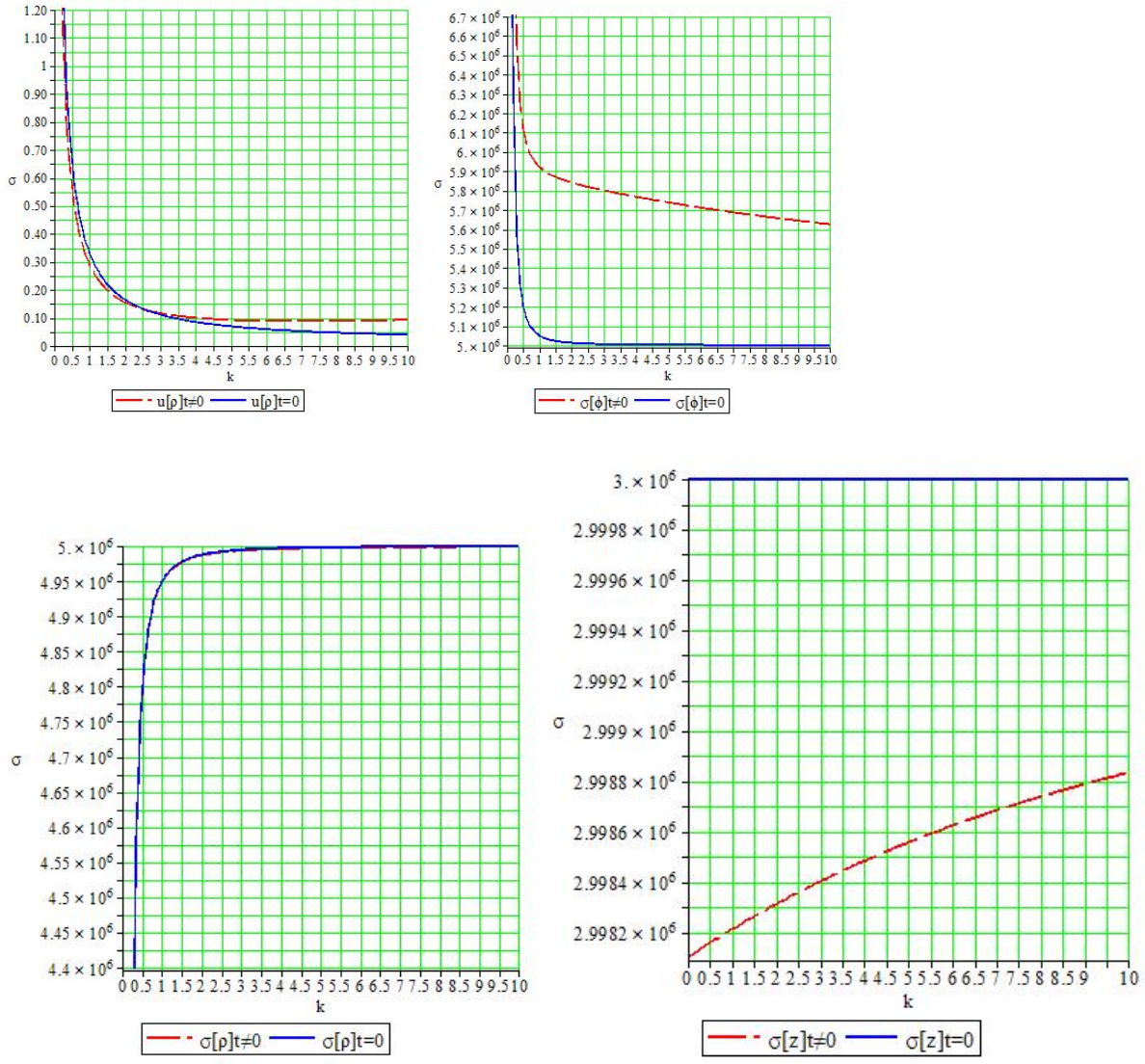


Figure 2. The graphs of radial displacement and normal stresses

Conclusion. Based on the conducted analyses, it was determined that accounting for the temperature function $T(\rho)$ in a thick-walled pipe significantly affects its stress-strain state. The graphical results show that all primary mechanical quantities are distributed non-uniformly across the radius, and their values are directly dependent on the temperature field.

Radial displacement u reaches its maximum value on the inner surface of the pipe and decreases monotonically as the radius increases. When thermal effects are considered, additional deformations arise due to thermal expansion, leading to a notable difference, particularly in the inner layers. Thus, the deformation state depends on the temperature gradient.
The circumferential stress σ_ϕ takes its highest value at the inner wall and decreases towards the outer surface. In the presence of temperature effects, a redistribution of stresses is observed. Due to the magnitude of the temperature gradient in the inner layers, thermal stresses combine with mechanical stresses, altering the overall stress level. This situation directly impacts the strength and reliability parameters of the pipe.
As seen from the graph, the lines for radial stress σ_ρ overlap for cases both with and without consideration of the temperature function. This indicates that radial stress in this problem is not significantly dependent on temperature. Therefore, the thermal component in the radial direction does not change the overall stress distribution.
Normal stress σ_z remains constant along the radius when temperature is not considered. However, when the temperature function is introduced, the stress value decreases and forms a curved distribution across the radius. This implies that thermal deformations significantly influence the stress state in the axial direction.

In general, the temperature field significantly affects both the quantitative values and the distribution patterns of stresses in a thick-walled pipe. The most substantial differences are observed in the inner layers. Therefore, accounting for the temperature function is a necessary condition in the strength calculation of thick-walled pipes. Otherwise, there is a risk of miscalculating the actual stress values in the inner layers.

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