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UCH O‘LCHOVLI FAZODA ISSIQLIK TARQALISH TENGLAMASI UCHUN IKKI NUQTALI TESKARI MASALANING KORREKLIGI HAQIDA

Annotatsiya: Ushbu maqolada uch o‘lchovli fazoda issiqlik tarqalishi tenglamasi uchun nolakal chegaraviy to‘g‘ri va teskari masalalarning umumlashgan yechimini Sobolev fazosida mavjud va yagonaligi o‘rganilgan.

Kalit so‘zlar: Parallelepiped, funksiya, differensial tenglamalar, matritsa, sohada, hosila.

О КОРРЕКТНОСТИ ДВУХТОЧЕЧНОЙ ОБРАТНОЙ ЗАДАЧИ ДЛЯ УРАВНЕНИЯ РАСПРЕДЕЛЕНИЯ ТЕПЛА В ТРЕХМЕРНОМ ПРОСТРАНСТВЕ

Аннотация: В данной статье исследуются существование и единственность обобщенного решения нелокальных краевых прямых и обратных задач для уравнения диффузии тепла в трехмерном пространстве в пространстве Соболева.

Ключевые слова: Параллелепипед, функция, дифференциальные уравнения, матрица, область определения, производная.

ON THE CORRECTNESS OF THE TWO-POINT INVERSE PROBLEM FOR THE EQUATION OF HEAT DISTRIBUTION IN THREE- DIMENSIONAL SPACE

Annotation: In this article, the existence and uniqueness of the generalized solution of nonlocal boundary value direct and inverse problems for the equation of heat diffusion in three-dimensional space in Sobolev space is studied.

Keywords: Parallelepiped, function, differential equations, matrix, Ω in the field, derivative.

Introduction: It is worth mentioning that in many scientific works, we emphasize that the boundary conditions have a local character and the methods of solving the equations are classical methods. However, the modern problems of natural science create the need to formulate and study qualitatively new problems, a clear example of which is the research of non-linear boundary problems. The relations connecting the value of the solution and its derivatives at the boundary and interior points of the sphere are called nonlocal problems. There are many open and understudied problems in the theory of nonlocal boundary value problems for differential equations of both theoretical and practical interest. Among them, for example, we note a wide category of questions related to the uniqueness, existence and stability (stationarity) of solutions. In recent decades, nonlocal problems for differential equations have been actively studied by many mathematicians. During the study of non-local issues, it was found that they are closely related to the opposite issues (management issues). In the theory of differential equations, such

problems are called inverse problems, in which the problem of determining the coefficients or the right side (external force) of the equation is understood along with determining the solution of the differential equation. The interest in studying inverse problems for mathematical physics equations is explained by the importance of their application in various fields of mechanics, seismology, medical tomography, and geophysics.

Analysis of literature on the topic of research: The main content and results of the research work at the conference “theoretical foundations and practical issues of modern mathematics” of Andijan State University on March 25-26, 2022. On April 21-22, 2022, the scientific and Practical Conference of young scientists on the topic “Mathematics, Mechanics and intellectual technologies” of the National University of Uzbekistan, named after. On April 29-30, 2022, in the city of Dushanbe, it was expressed in the conference called "Modern problems of the number theory of mathematical analysis". The general approach to solving incorrect problems was developed by A. N. Tikhonov, and A. N. Tikhonov, J. Lyons, Lattes, M. M. Lavrentiev, A. S. Alekseev, V. A. Morozov, V. Ya Arsenin, V. G. Romanov, S. I. Kabanikhin, B. A. Bubnova, A. Kojanov, K. S. Developed by Fayazova, D. Durdiev and others. Two-point inverse problems for mathematical physics equations have been studied very little.

$G=(0,T)\times(0,1)\times(0,l)=Q\times(0,l)\subset R^3$ Let's look at the following equation of heat distribution in a field in the form of a parallelepiped.

$$Lu=u_t-\Delta u+c(x,t)u=g(x,t,y)+\sum_{k=1}^{2}h_k(x,t)f_k(x,t,y), \qquad (3.2.1)$$

Here $\Delta u=u_{xx}+u_{yy}$, $g(x,t,y),$ and $f_k(x,t,y),(k=1,2.)$ - given functions. We consider the coefficients of equation (3.2.1) as sufficiently smooth functions. Two-point linear inverse problem. equation (3.2.1) in the field and the following boundary conditions

$$\gamma u\big|_{t=0}=u\big|_{t=T}, \qquad (3.2.2)$$

$$u\big|_{x=0}=u\big|_{x=1}=0, \qquad (3.2.3)$$

$$u\big|_{y=0}=u\big|_{y=l}=0, \qquad (3.2.4)$$

And additional conditions

$$u\big|_{y=\ell_k}=\varphi_k(x,t),k=1,2,\ 0<\ell_1<\ell_2<\ell<+\infty. \qquad (3.2.5,\ 3.2.6)$$

Satisfactory and belongs to the following class

$U=\{(u,h_k,k=1,2)\in W_2^{2,1}(G),\ D_y^3(u_t,u_x,u_{xx})\in L_2(G),\ D_y^4u\in L_2(G);\ h_k\in W_2^2(Q)\}.$

(u,h_1,h_2) find functions. We introduce the following definitions $g_i(x,t)=g(x,t,l_i),$

$f_{ij}(x,t)=f_i(x,t,l_j),\ \forall i,j=1,2.\ F=\{f_{ij}\}_{i,j=1}^{2}$ elements through $f_{ij}(x,t)=f_i(x,t,\ell_j)$ we define the second, ordered square matrix consisting of functions, i.e

$$F=\begin{pmatrix} f_{11} & f_{21} \\ f_{12} & f_{22} \end{pmatrix},\quad \Delta=\det F=\begin{vmatrix} f_{11} & f_{21} \\ f_{12} & f_{22} \end{vmatrix}$$ we define its determinant by

Theorem. We assume that the following conditions are met for the coefficients of equation (1). $\lambda c-c_t\geq\delta>0,$ optional $(x,t)\in\overline{Q},$ for s, here

$\lambda=\frac{2}{T}\ln|\gamma|>0,\ |\gamma|>1,\ |\det F|\geq\varepsilon>0;\ \varphi_i,\varphi_{it},\varphi_{ixx}\in W_2^2(Q_1);\ \gamma\varphi_i\big|_{t=0}=\varphi_i\big|_{t=T};$
$\varphi_i\big|_{x=0}=\varphi_i\big|_{x=1}=0;\ g_i\in W_2^2(Q);\ f_{ij}\in W_2^2(Q);\ \forall_{i,j}=1,2$ and we assume $\beta\equiv M\sum_{k=1}^{2}\left\|(1+D_y^3)f_k\right\|^2_{W_2^1(Q)}<1,$ let the inequality be fulfilled, here $M=const(mes(Q),\det F).$ Then it's optional g and f_i the following conditions $(1+D_y^3)f_i\in W_2^2(G);\ f_i\big|_{x=0}=f_i\big|_{x=1}=0,\ \forall i=1,2\ (1+D_y^3)g\in W_2^2(G);\ g\big|_{x=0}=g\big|_{x=1}=0,$ the solution of the problem (3.2.1)-(3.2.5) is given for satisfying functions

$U-$ is available and unique in the class.

The unknown functions are defined as follows.

$$u(x,t,y)=\sqrt{\frac{2}{\ell}}\cdot\sum_{k=1}^{\infty}u_k(x,t)\sin\mu_k y,\ \mu_k=\frac{k\pi}{\ell},\ k=1,2,3....$$

$$h_1=\frac{\begin{vmatrix}\Phi_1 & f_{21}\\ \Phi_2 & f_{22}\end{vmatrix}}{\begin{vmatrix}f_{11} & f_{21}\\ f_{12} & f_{22}\end{vmatrix}}=\frac{1}{\Delta}[\Phi_1 f_{22}-\Phi_2 f_{21}]\ h_2=\frac{\begin{vmatrix}f_{11} & \Phi_1\\ f_{12} & \Phi_2\end{vmatrix}}{\begin{vmatrix}f_{11} & f_{21}\\ f_{12} & f_{22}\end{vmatrix}}=\frac{1}{\Delta}[\Phi_2 f_{11}-\Phi_1 f_{12}]$$

$$\Phi_i=\varphi_{it}-\varphi_{ixx}+c\varphi_i+\sum_{k=1}^{\infty}\mu_k^2u_k\sin\mu_k\ell_i=L_0\varphi_i+\sum_{k=1}^{\infty}\mu_k^2u_k\sin\mu_k\ell_i,\quad i=1,2.$$

here

$u_k(x,t)$ functions Q is found from the semi-indefinite problem set in the following system of loaded infinite parabolic equations in the field.

$$Lu_k = u_{kt} - u_{kxx} + (c+\mu_k^2)u_k = \frac{f_{1k}}{\Delta}[f_{22}(L_0\varphi_1 + \sum_{m=1}^{\infty}\mu_m^2 u_m \sin\mu_m\ell_1) - f_{21}(L_0\varphi_2 + \sum_{m=1}^{\infty}\mu_m^2 u_m \sin\mu_m\ell_2)] +$$

$$\frac{f_{2k}}{\Delta}[f_{11}(L_0\varphi_2 + \sum_{m=1}^{\infty}\mu_m^2 u_m \sin\mu_m\ell_2) - f_{12}(L_0\varphi_1 + \sum_{m=1}^{\infty}\mu_m^2 u_m \sin\mu_m\ell_1)] \quad (3.2.7)$$

$$\gamma u_k\big|_{t=0} = u_k\big|_{t=T}, \quad (3.2.8)$$

$$u_k\big|_{x=0} = u_k\big|_{x=1} = 0, \quad (3.2.9)$$

here

$$f_1(x,t,y) = \sum_{k=1}^{\infty} f_{1k}(x,t)\sin\mu_k y, \quad f_1(x,t,\ell_1) = f_{11}(x,t) = \sum_{k=1}^{\infty} f_{1k}(x,t)\sin\mu_k\ell_1, \; f_2(x,t,\ell_1) = f_{21}(x,t) = \sum_{k=1}^{\infty} f_{2k}(x,t)\sin\mu_k\ell_1,$$

$$f_{jk} = \sqrt{\frac{2}{\ell}}\int_0^{\ell} f_j \sin\mu_k y dy, \forall j = 1,2; \; g_k = \sqrt{\frac{2}{\ell}}\int_0^{\ell} g\sin\mu_k y dy, \; k = 1,2,3,\ldots$$

$L_0\varphi_i = \varphi_{it} - \varphi_{ixx} + c\varphi_i - g_i, \quad i = 1,2.$ (parabolic operators)

We prove the theorem in several stages: First of all $u(x,t,y)$ function $u\big|_{y=\ell_i} = u(x,t,\ell_i) = \varphi_i(x,t), i = 1,2$ (3.2.5, 3.2.6) we show that it satisfies additional conditions. For this we assume the opposite. Suppose such a function $\vartheta_i(x,t)$ exists, let the following condition be relevant for it $u\big|_{y=\ell_i} = \vartheta_i(x,t) \neq \varphi_i(x,t)$, i.e

$$u\big|_{y=\ell_i} = \sum_{k=0}^{\infty} u_k(x,t)\sin\mu_k\ell_i = \vartheta_i(x,t) \neq \varphi_i(x,t),$$

Then it's a new function $z_i(x,t) = \vartheta_i(x,t) - \varphi_i(x,t) \Rightarrow \vartheta_i = z_i + \varphi_i$ for, Ω using problems (3.2.7)-(3.2.9) in the field, solve both sides of the equation (3.2.7) $\sin\mu_k\ell_i$ from 1 by multiplying by ∞ summing up, we get the following loaded equation.

$$\sum_{k=1}^{\infty} Lu_k \sin \mu_k \ell_i = \sum_{k=1}^{\infty} u_{kt} \sin \mu_k \ell_i - \sum_{k=1}^{\infty} u_{kxx} \sin \mu_k \ell_i + \sum_{k=1}^{\infty} (c+\mu_k^2) u_k \sin \mu_k \ell_i =$$

$$= \frac{\sum_{k=1}^{\infty} f_{1k} \sin \mu_k \ell_i}{\Delta} [\Phi_1 f_{22} - \Phi_2 f_{21}] + \frac{\sum_{k=1}^{\infty} f_{2k} \sin \mu_k \ell_i}{\Delta} [\Phi_2 f_{11} - \Phi_1 f_{12}] =$$

$$\frac{\sum_{k=1}^{\infty} f_{1k} \sin \mu_k \ell_i}{\Delta} [f_{22}(L_0\varphi_1 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_1) - f_{21}(L_0\varphi_2 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_2)] +$$

$$\frac{\sum_{k=1}^{\infty} f_{2k} \sin \mu_k \ell_i}{\Delta} [f_{11}(L_0\varphi_2 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_2) - f_{12}(L_0\varphi_1 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_1)] = \quad (3.2.10)$$

$$\frac{f_{1i}}{\Delta} [f_{22}(L_0\varphi_1 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_1) - f_{21}(L_0\varphi_2 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_2)] +$$

$$\frac{f_{2i}}{\Delta} [f_{11}(L_0\varphi_2 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_2) - f_{12}(L_0\varphi_1 + \sum_{m=1}^{\infty} \mu_m^2 u_m \sin \mu_m \ell_1)]$$

From formula (3.2.10) to make it easier for the first time $i=1$ let's look at the situation, taking into account the designations, we create the following formula from the formula (3.2.10).

$$\vartheta_{1t} - \vartheta_{2xx} + c\vartheta_1 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1 = \frac{f_{11}}{\Delta} [f_{22}(L_0\varphi_1 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1) - f_{21}(L_0\varphi_2 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_2)] +$$

$$+ \frac{f_{21}}{\Delta} [f_{11}(L_0\varphi_2 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_2) - f_{12}(L_0\varphi_1 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1)] = \frac{L_0\varphi_1}{\Delta} [f_{11}f_{22} - f_{11}f_{21}] +$$

$$\frac{\sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1}{\Delta} [f_{11}f_{22} - f_{12}f_{21}] + \frac{L_0\varphi_2}{\Delta} [f_{21}f_{11} - f_{21}f_{11}] + \frac{\sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_2}{\Delta} [f_{21}f_{11} - f_{21}f_{11}] =$$

$$= L_0\varphi_1 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1 \quad (3.2.11)$$

Then it's a new function $z_i(x,t) = \vartheta_i(x,t) - \varphi_i(x,t) \Rightarrow \vartheta_i = z_i + \varphi_i$ because $\mathcal{Q}$ using the problems (3.2.7)-(3.2.9) in the field, we form the following equation from the formula (3.2.11).

$$L_0(z_1 + \varphi_1) + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1 = L_0\varphi_1 + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_1$$

(3.2.12)

from equality (3.2.12) and conditions (3.2.8), (3.2.9) to the given function in the theorem $\phi(x,t)$ Based on the given conditions, we form the following problem.

$$L_0 z_1 = 0 \quad (3.2.13)$$

$$\gamma z_1\big|_{t=0} = z_1\big|_{t=T}, \quad (3.2.14)$$

$$z_1\big|_{x=0} = z_1\big|_{x=1} = 0, \qquad (3.2.15)$$

Our task now is to show that problem (3.2.13)-(3.2.15) has a unique solution. For this, the following equation $2(L_0 z_1, e^{-\lambda t} z_{1t}) = 0$ Integrating by pieces, conditions of the theorem, limit (3.2.14),(3.2.15) conditions and $|\gamma| > 1,$ taking into account, we form the following inequality $\|z_1\|_{W_2^1(Q)} \le 0,$ from here $z_1(x,t) = 0.$ Thus, since the problem (3.2.13)-(3.2.15) has a unique solution $\vartheta_1(x,t) \equiv \varphi_1(x,t),$ originates. Hence the condition of problem (3.2.5). $u(x,t,\ell_1) = \varphi_1(x,t).$ done. Also, condition (3.2.6) of the problem, i.e $u(x,t,\ell_2) = \varphi_2(x,t).$ execution is shown. Our task now is to prove that the solution of the problem (3.2.7)-(3.2.9) has a unique solution. For this Q we use successive approximations, Galerkin and a priori estimation methods for the system of heat diffusion equations loaded in the field.

$$Lu_k^{(l)} = u_{kt}^{(l)} - u_{kxx}^{(l)} + (c+\mu_k^2)u_k^{(l)} = \frac{f_{1k}}{\Delta}[f_{22}(L_0\varphi_1 + \sum_{m=1}^{\infty}\mu_m^2 u_m^{(l-1)}\sin\mu_m\ell_1) - f_{21}(L_0\varphi_2 + \sum_{m=1}^{\infty}\mu_m^2 u_m^{(l-1)}\sin\mu_m\ell_2)] +$$

$$\frac{f_{2k}}{\Delta}[f_{11}(L_0\varphi_2 + \sum_{m=1}^{\infty}\mu_m^2 u_m^{(l-1)}\sin\mu_m\ell_2) - f_{12}(L_0\varphi_1 + \sum_{m=1}^{\infty}\mu_m^2 u_m^{(l-1)}u_m\sin\mu_m\ell_1)] = F_k(u_k^{(l-1)})$$

(3.2.16)

$$\gamma u_k^{(l)}\big|_{t=0} = u_k^{(l)}\big|_{t=T}, \qquad (3.2.17)$$

$$u_k^{(l)}\big|_{x=0} = u_k^{(l)}\big|_{x=1} = 0, \qquad (3.2.18)$$

To investigate this problem (3.2.16)-(3.2.18) and prove the theorem, we need the following definitions, spaces and auxiliary lemmas.

It has the following finite norm

$$\langle\vartheta_k\rangle_i^2 = \begin{cases} \text{если} \quad i=0, \quad то \quad \sum_{k=1}^{\infty}(1+\mu_k^6)\|\vartheta_k\|^2_{W_2^0(Q)}, W_2^0(Q) = L_2(Q) \\ \text{если} \quad i=1, \quad то \quad \sum_{k=1}^{\infty}(1+\mu_k^6)\|\vartheta_k\|^2_{W_2^1(Q)} \\ \text{если} \quad i=2, \quad то \quad \langle\vartheta_k\rangle_1^2 + \langle\vartheta_{kxx}\rangle_0^2 \end{cases} \qquad (C)$$

we introduce the space of vector functions $W_i(Q) = \{\vartheta_k \,|\, \vartheta_k \in W_2^i(Q),\ i = 0,1,2:\ k = 1,2,\ldots\}$.

It can be seen that finite (C) has a norm $W_i(Q)$ the space will be a Banach space. $W_2^i(Q), i = 0,1,2$ according to the definition, the following relationship is appropriate. $W_2(Q) \subset W_1(Q) \subset W_0(Q)$

Now $W(Q)$ through (3.2.17),(3.2.18) satisfying the conditions $\{u_k(x,t)\}_{k=1}^{\infty} \in W_2(Q)$ we introduce the space of vector functions.

Description. (3.2.16)-(3.2.18) as the solution of the problem, satisfying the equation (3.2.16), $\{u_k(x,t)\}_{k=1}^{\infty} \in W(Q)$ we say to the vector-function.

Lemma 1. Let's assume that all conditions of the lemma are fulfilled. In that case, the following a priori estimates are appropriate for solving the problem (3.2.16) - (3.2.18).

I) $\left\langle u_k^{(l)} \right\rangle_1^2 \le const(\beta_0, f) < +\infty$

II) $\left\langle u_k^{(l)} \right\rangle_2^2 \le const(\beta_0, f) < +\infty,$ here $\beta_0 = C\left(\left\langle g_k \right\rangle_0^2 + \sum_{j=1}^{n-1} \left\| \Phi_i \right\|_0^2 \left\langle f_{jk} \right\rangle_0^2 \right)$

Proof. We prove the first a priori estimate:

For this we consider the following equation:

$$(Lu_k^{(l)}, e^{-\lambda t} u_{kt}^{(l)})_0 = (F_k(u_k^{(l-1)}), e^{-\lambda t} u_{kt}^{(l)})_0 \qquad (3.2.19)$$

Integrating equality (3.2.19) piecewise, using the conditions of the theorem and boundary conditions (3.2.17), (3.2.18) and Cauchy's inequality, we form the first estimate.

Now we prove the second a priori estimate:

For this we consider the following equation:

$$-(Lu_k^{(l)}, e^{-\lambda t} u_{kxx}^{(l)})_0 = -(F_k(u_k^{(l-1)}), e^{-\lambda t} u_{kxx}^{(l)})_0 \qquad (3.2.20)$$

Integrating equality (3.2.20) piecewise, using the conditions of the theorem and boundary conditions (3.2.17), (3.2.18) and Cauchy's inequality, we form the second estimate.

Now $W(Q)$ we define a new function in space: $\vartheta_k^{(l)} = u_k^{(l)} - u_k^{(l-1)}$

Lemma 2. Suppose that all the conditions of theorem and theorem 1 are fulfilled, in that case $\{\vartheta_k^{(l)}\}$ the following a priori estimates for the functions are appropriate.

III) $\left\| \vartheta_k^{(l)} \right\|_1^2 \le \beta_1^{(l+1)} const(f)$

IV) $\left\| \vartheta_k^{(l)} \right\|_2^2 \le \beta_1^{(l+1)} const(f)$, here $\beta_1 \equiv M \cdot \sum_{j=1}^{n-1} \left\| (1 + D_y^3) f_j \right\|_{W_2^1(Q)} < 1$

Proof. From problems (3.2.16)–(3.2.18). $Q-$ in the field $\{\vartheta_k^{(l)}\}$ we form the following problems for the functions.

$$L\vartheta_k^{(l)} = \vartheta_{kt}^{(l)} - \vartheta_{kxx}^{(l)} + (c+\mu_k^2)\vartheta_k^{(l)} = \frac{f_{1k}}{\Delta}[f_{22}\sum_{m=1}^{\infty}\mu_m^2\vartheta_m^{(l-1)}\sin\mu_m\ell_1 - f_{21}\sum_{m=1}^{\infty}\mu_m^2\vartheta_m^{(l-1)}\sin\mu_m\ell_2] +$$

$$\frac{f_{2k}}{\Delta}[f_{11}\sum_{m=1}^{\infty}\mu_m^2\vartheta_m^{(l-1)}\sin\mu_m\ell_2 - f_{12}\sum_{m=1}^{\infty}\mu_m^2\vartheta_m^{(l-1)}\sin\mu_m\ell_1] = F_k(\vartheta_k^{(l-1)}) \qquad (3.2.21)$$

$$\gamma\vartheta_k^{(l)}\big|_{t=0} = \vartheta_k^{(l)}\big|_{t=T}, \qquad (3.2.22)$$

$$\vartheta_k^{(l)}\big|_{x=0} = \vartheta_k^{(l)}\big|_{x=1} = 0, \qquad (3.2.23)$$

$\{u_k^{(l)}\}$ since the conditions of lemma 1 are reasonable for the functions, we repeat the methods of obtaining the first and second values $\{\vartheta_k^{(l)}\}$ we generate a priori estimates for functions III) and IV).

Now $W(Q)$ radius in space $R-$ was $B_R(Q)$ we get a ball.

$B_R(Q) = \left\{\forall\vartheta_k^{(l)} \mid \vartheta_k^{(l)} \in W(Q);\ \left\|\vartheta_k^{(l)}\right\|_2^2 \le R\right\}$, here $R-$ some constant number.

Lemma 3. Suppose that all conditions of lemmas 1 and 2 are fulfilled, then (3.2.16)–(3.2.18) is the problem $B_R(Q)$ will have a unique solution on the sphere.

Proof. $B_R(Q)$ we define the following operator (reflection) on the sphere:

$$u_k^{(l)} = L^{-1}F_k(u_k^{(l-1)}) = Pu_k^{(l-1)}$$

$\{u_k^{(l)}\}$ Since the conditions of Lemma 1 and Lemma 2 are valid for vector functions:

1) P – operator $B_R(Q)$ reflects the sphere to itself with one value.
2) P – there will be a compression reflection.

Based on the principle of compressive reflections, we note that problem (3.2.6) - (3.2.8) $B_R(Q)$ will have a unique solution on the sphere.

$B_R(Q)$ in $\{u_k\}$ using Parseval-Steklov equality for functions (3.2.1)-(3.2.3) $u(x,t,y)$ we get the solution from the class. The theorem is proved.

Result-1. If given $\varphi_i(x,t)$ if we consider the function as a solution to the following problem $\varphi_i \in W_{2,x,t}^{4,3}, \quad g_i \in W_2^2$

$$L\varphi_i = \varphi_{it} - \varphi_{ixx} - g_i = 0 \qquad (3.2.24)$$

$$\gamma\, \varphi_i\big|_{t=0} = \varphi_i\big|_{t=T}, \qquad (3.2.25)$$

$$\varphi_i\big|_{x=0} = \varphi_i\big|_{x=1} = 0, \qquad (3.2.26)$$

In that case $h_1 = \frac{1}{\Delta}[\Phi_1 f_{22} - \Phi_2 f_{21}],\ h_2 = \frac{1}{\Delta}[\Phi_2 f_{11} - \Phi_1 f_{12}]$

here

$$\Phi_i = \varphi_{it} - \varphi_{ixx} + c\varphi_i - g_i + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_i = L_0 \varphi_i + \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_i = \sum_{k=1}^{\infty} \mu_k^2 u_k \sin \mu_k \ell_i, \quad i = 1,2.$$

and the proof of the theorem can be much simplified.

Result-2. For equation (3.2.1), replace condition (3.2.2) in problem Koshi $u\big|_{t=0} = u_0(x)$ condition, we can get the same result.

Summary: we conclude that in this work, the correctness (having a unique solution) of the generalized solutions of Cauchy and nonlocal problems for the string vibration equation in Sobolev spaces was studied using modern methods, methods of a priori estimates.

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