ISSN:3030-332X
Impact factor: 9,493
International journal of scientific researchers

BUXORO DAVLAT UNIVERSITETI DIFFERENSIAL TENGLAMALARI KAFEDRASI O‘QITUVCHISI

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Teskari matrisa tatbiqi

Annotatsiya. Ushbu material talabalar va chiziqli algebrani o'rganuvchilar uchun matrisalar hisobini ham analitik (formulalar orqali), ham algoritmic (elementar almashtirishlar orqali) usullarda chuqur o'zlashtirishga xizmat qiladi.

Kalit so’zlar: Matrisalar hisobi, chiziqli algebra, analitik usullar, algoritmik usullar, elementar almashtirishlar, talabalar uchun chiziqli algebra.

Annotation. This material is designed for students and learners of linear algebra, serving to help them deeply master matrix calculus through both analytical (formula-based) and algorithmic (elementary transformation-based) methods.

Key words: Matrix calculus, linear algebra, analytical methods, algorithmic methods, elementary transformations, linear algebra for students.

Teskari matrisa

A – matritsa n-chi tartibli kvadrat matritsa bo’lsin. A matritsa uchun AB=BA=E tenglikni qanoatlantiruvchi B matritsa A ga teskari matritsa deyiladi va u $B = A^{-1}$ ko’rinishda belgilanadi. Teskari matritsa elementlarini

$$b_{ij} = \frac{A_{ji}}{\det A}$$

formula yordamida topish mumkin, bunda A_{ji} lar a_{ji} elementlarning algebraik to’ldiruvchisidir. A matritsa teskarilanuvchi deb aytiladi, agar $\det A \neq 0$, ya’ni A matritsa xosmas bo’lsa.

Har qanday xosmas A matritsani faqat satrlar (yoki faqat ustunlar) elementar almashtirishlari yordamida birlik matritsaga keltirish mumkin. Elementar almashtirishlarni xuddi shunday ketma-ketlikda E birlik matritsaga tadbiq qilsak, teskari matritsa A^{-1} ni hosil qilamiz. A va E matritsalarni chiziq yordamida qo’shni yozib ular ustida elementar almashtirishlarni bir vaqtda bijarish juda qulaydir.

Teskari matritsani hisoblash $AX = B, \quad YA = B$ matritsaviy tenglamalarni yechish bir-biri bilan bog’langandir, bunda A, B – berilgan matritsalar, X, Y – izlanayotgan noma’lum matritsalar. Agar A matritsa to’g’ri burchakli matritsa yoki xosmas matritsa bo’lsa, matritsaviy tenglamalarni yechish X matritsaning har bir ustuni yoki Y matritsaning har bir satri elementlari uchun hosil bo’ladigan chiziqli tenglamalar sistemasini yechishga keltiriladi. Bu tenglamalarni hosil qilish uchun tenglamaning har ikkala tomonidagi matritsalarning mos elementlarini bir-biriga tenglashtirish lozim. Agar A matritsa xosmas bo’lsa, matritsaviy tenglamalarning yechimlari qyidagi formulalar yordamida topiladi:

$$X = A^{-1}B, \quad Y = BA^{-1}.$$

1-m i s o l. Berilgan matritsa uchun teskari matritsa topilsin

2-

$$A = \begin{matrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{matrix}.$$

Yechish. $\det A = -1$ bo'lganligi uchun teskari matritsa A^{-1} mavjud. Matritsa elementlarining algebraik to'ldiruvchilarini topamiz:

$$A_{11}=8,\quad A_{21}=-29,\quad A_{31}=11,$$
$$A_{12}=5,\quad A_{22}=-18,\quad A_{32}=7,$$
$$A_{13}=-1,\quad A_{33}=3,\quad A_{33}=-1.$$

Teskari matritsani topish formulasiga asosan bu algebraik to'ldiruvchilarni (-1) ga bo'lib, teskari matritsani hosil qilamiz:

$$A^{-1} = \begin{matrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{matrix} .\blacksquare$$

2-m i s o l. Satrlarning elementar almashtirishlari yordamida teskari matritsa A^{-1} ni toping

$$A = \begin{matrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{matrix} .$$

Yechish. Quyidagilarni hosil qilamiz:

$$\left(\begin{array}{ccc|ccc} 3 & -4 & 5 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 3 & -5 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & -1 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 3 & -5 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2-2R_1}$$

$$\xrightarrow{R_2-2R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & -1 & 0 \\ 0 & -1 & -7 & -2 & 3 & 0 \\ 3 & -5 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3-3R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & -1 & 0 \\ 0 & -1 & -7 & -2 & 3 & 0 \\ 0 & -2 & -13 & -3 & 3 & 1 \end{array}\right) \xrightarrow{R_3-2R_2}$$

$$\xrightarrow{R_3-2R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & -1 & 0 \\ 0 & -1 & -7 & -2 & 3 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right) \xrightarrow{R_1-4R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 3 & 11 & -4 \\ 0 & -1 & -7 & -2 & 3 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right) \xrightarrow{R_2+7R_1}$$

$$\xrightarrow{R_2+7R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 11 & -4 \\ 0 & -1 & 0 & 5 & -18 & 7 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & 29 & -11 \\ 0 & -1 & 0 & 5 & -18 & 7 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right) \xrightarrow{(-1)R_2}$$

$$\xrightarrow{(-1)R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & 29 & -11 \\ 0 & 1 & 0 & -5 & 18 & -7 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array}\right) .$$

Bunda R_i matritsaning i-satri.
Shunday qilib, teskari matritsa quyidagi ko'riniga ega bo'ladi:

$$A^{-1} = \begin{matrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{matrix} .\blacksquare$$

3-m i s o l. Quyidagi tenglamalardan X matritsani toping:

$$\begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix} X \begin{matrix} -4 & 0 \\ 2 & 1 \end{matrix} = \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix}.$$

Yechish. $\det A = -1 \quad 0$ va $\det B = -4 \quad 0$ bo'lgani uchun A^{-1} va B^{-1} teskari matritsalar mavjud. Tenglamaning chap va o'ng tomonlarini chapdan A^{-1} ga, o'ngdan B^{-1} ga ko'paytirsak, quyidagi tenglikka ega bo'lamiz:

$$A^{-1}(AXB)B^{-1} = A^{-1}CB^{-1}.$$

$$A^{-1}(AXB) = (A^{-1}A)X(BB^{-1}).$$

$A^{-1}A = E, BB^{-1} = E$ va $EXE = X$ bo'lganligi uchun $X = A^{-1}CB^{-1}$ kelib chiqadi.
Bu tenglikka asosan X matritsani hisoblaymiz:

$$A^{-1} = \begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}, \quad B^{-1} = \begin{matrix} -0{,}25 & 0 \\ 0{,}5 & 1 \end{matrix},$$

$$X = \begin{matrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{matrix} \begin{matrix} -0{,}25 & 0 \\ 0{,}5 & 1 \end{matrix} = \begin{matrix} -0{,}5 & 1 \\ 0{,}5 & 1 \end{matrix}. \blacksquare$$

4-m i s o l. Har qanday qanday elementar almashtirishlar xosmas matritsalarning ko'paytmasidan iborat ekanligini isbotlang.

Yechish. A matritsa satrlarining har qanday elementar almashtirishlari A matritsani chapdan elementar matritsaga ko'paytirishdan iborat bo'lib, bu elementar matritsani birlik matritsadan o'shanday elementar almashtirish yordamida hosil qilish mumkin. Xosmas matritsani elementar almashtirishlar yordamida birlk matritsaga keltirish mumkin. Shuning uchun $S_k \quad S_1 A = E,$ quyidagiga ega bo'lamiz:

bundan esa $S_k \quad S_1 = A^{-1}, \quad A = S_1^{-1} \quad S_k^{-1}$. $S_1^{-1},\ldots,S_k^{-1}$ matritsalar hamda $S_1,\ldots,S_k$ matritsalar ham elementar matritsalardir, ularni birlik matritsadan satrlarning «teskari» elementar almashtirishi yordamida hosil qilish mumkin. ■

5-m i s o l. Berilgan matritsani elementar matritsalarning ko'paytmasiga yoying: $\begin{matrix} 1 & 1 \\ 0 & -2 \end{matrix}$.

Yechish. To'rtinchi misolning yechimiga asosan bunda $A = S_1^{-1} \quad S_k^{-1}$ matritsalar A matritsa satrlarining elementar almashtirishlariga mos keladiki, uni birlik matritsaga keltiradi. $S_1,\ldots,S_k$ matritsalarni tanlab keyin $S_1^{-1},\ldots,S_k^{-1}$ larni topamiz. Bu jarayonni bitta qadamga kamaytirish mumkinligini ko'rsatamiz. $A = \begin{matrix} 1 & 1 \\ 0 & -2 \end{matrix}$ matritsani soddalashtiramiz. Matritsaning ikkinchi satrini $-\frac{1}{2}$ ga ko'paytirish A ni chap tomondan $\begin{matrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{matrix}$ matritsaga ko'paytirish bilan teng kuchlidir. Quyidagi tenglik hosil bo'ladi:

$$\begin{matrix}1 & 0\\ 0 & -\frac{1}{2}\end{matrix}\begin{matrix}1 & 1\\ 0 & -2\end{matrix}=\begin{matrix}1 & 1\\ 0 & 1\end{matrix}=B \qquad (*)$$

B matritsa elementar matritsadir. Hisoblaymiz:

$$\begin{matrix}1 & 0\\ 0 & -\frac{1}{2}\end{matrix}^{-1}=\begin{matrix}1 & 0\\ 0 & -2\end{matrix}=S$$

(*) tenglikning har ikkala tomonini chapdan S ga ko'paytirsak, izlanayotgan yoyilma kelib chiqadi:

$$\begin{matrix}1 & 1\\ 0 & -2\end{matrix}=SB=\begin{matrix}1 & 0 & 1 & 1\\ 0 & -2 & 0 & 1\end{matrix}.\blacksquare$$

6-m i s o l. A va B matritsalar satrlarining elementar almashtirishlari yordamida $A^{-1}B$ ko'paytmani hosil qilish usulini keltiring.

Yechish. A va B matritsalarni ketma-ket yozamiz. (AB) matritsaning satrlari bilan elementar almashtirishlarni bajaramiz. Bu almashtirishlar A matritsani E matritsaga keltirsin. U holda bu almashtirishlar natijasida A matritsa joyida E matritsa, B matritsa joyida A^{-1} matritsa hosil bo'ladi.

Haqiqatdan ham, $S_1 \quad S_k A = E$ bo'lgani uchun $S_1 \quad S_k = A^{-1}$ bo'ladi. U holda $S_1 \quad S_k B = A^{-1}B$ (4 misolga qarang).

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