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THE ISSUE OF ESCAPE IN LINEAR DIFFERENTIAL GAMES

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Annotation: In these target areas, the movements of the chaser and hugs played a game given by different equations drawn in a simple way.

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Introduction: In the space Rk(k^2), a differential game R of n+m persons is considered: n pursuers Pi,...,Pn and m fleeing E\,..., Em [1-7]. The law of motion of each of the pursuers Pi has the form Xi = A(t)xi + Ui, Xi(to) = x0, x € V.The law of motion of each of the escaping Ej has the form yj = A(t)yj + Vj, yj (to) = yj Vj e V, where Xi, yj, Ui, Vj € Rk, A(t) is a continuous matrix function, V is a strictly convex compact with a smooth boundary, zj = x 0 — y0.

In [1, 2], sufficient conditions for the possibility of escape are given for the case of dim M ≤ n − 2. (Note that in the cases dim M =n − 1, n, the escape problem in the formulation of works [1, 2] is trivial). They consist of the condition of maneuverability and the condition of superiority of the fleeing over the pursuer.

The maneuverability condition of work [1] is formulated as follows.Let L be the orthogonal complement of M, l = dim L, πL : R n → L is the orthogonal projection on L. There exists a natural number k such that

a) each of the sets πL, Q, ΠCQ, . . . , ΠC k−2Q consists of a single point;
b) the set of nLC k−1Q is a body in L (i.e. it has points internal to L).

In the future, the conditions a), b) for brevity, we will call the Pontryagin-Mishchenko condition.The maneuverability condition of work [2] has the form.There exists a two-dimensional subspace W ⊂ L such that for no one-dimensional subspace W1 ⊂ W for all sufficiently small positive values τ there is no inclusion.

The condition of maneuverability of work formulated above [2] will be called the condition of rotatability in the future.In point B of this paper, an estimate of the number k in the condition is indicated Pontryagina-Mishchenko, in point B – a simple criterion for fulfilling the condition of rotatability.

M.S.Nikolsky showed that in the Pontryagin–Mishchenko condition, the number k cannot exceed the numbers n − 1 ([3]). N.Satimov noticed that the more precise inequality k ≤ n − 2l + 2 is true (for 2l ≤ n.If then, obviously, k = 1). The following theorem holds, giving an unimprovable estimate for the number k.Theorem 1. If R n, L, C, Q satisfy the Pontryaginamishchenko condition, then there is an inequality.

kl ≤ n. (3)

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If l = 0 or l = n, then the inequality (3) is obvious. Therefore , the assessment (3) we prove for 0 < l < n. First, simplify the conditions a), b).

Let S 0 is a subspace R n parallel to the carrier plane Q and of the same dimension with Q (i.e. if [Q] is the carrier plane Q, then S 0 = [Q] − Q). From condition b) it follows that

dim S 0 = dim Q ≥ l. (4)

If a is a point Q, internal with respect to [Q], then, obviously, α)Q

0 = Q − a ⊂ S 0

β) 0 is a point Q0 , internal with respect to S 0. Therefore, the linear shell Q0 coincides with S

0:[Q0] = S0. (5)

Further, from the condition α) it follows that the set ΠC iQ0 = πc iQ−ΠC ia for i ≤ k − 2 consists of a single point, and by virtue of β) this point is 0. Thus, the nLC iQ consists of zero. Hence, by virtue of (5) we get

πLS 0 = πLCS0 = . . . = πLC k−2S 0 = 0.

By definition of the projection, the ratio πLx = 0 is equivalent to the inclusion of x ∈ M, so it follows from the condition a) of the lemma that the subspaces S, CS, . . . , Ck−2S (i < k − 2) are contained in M. In addition, they are all l-dimensional, because

l = dim S ≥ dim CS ≥ . . . ≥ dim C k−2S ≥ dim C k−1S = l.

Consider the amount

Σ = S + CS + . . . + C k−2S ⊂ M. (6)

We show that in (6) the sum is a direct sum. To do this, assume that Σi = S + · · · + C iS (i < k − 2) is a direct sum (with i = 0, nothing needs to be assumed). Let N be a subspace Σi ∩ C i+1S. From N ⊂ Σi follows

C k−i−2N ⊂ C k−i−2S + · · · + C k−2S ⊂ M.

From N ⊂ C i+1S follows ¡ C k−i−2N ∉ ∩ M = 0. Therefore C k−i−2N = 0.From this equality and from the fact that the mapping C k−i−2 preserves the dimension of the subspace C i+1S it follows that N = 0. Thus, Σi ∩ C i+1S = 0, i.e. Σi+1 is also a direct sum. From the relation (6) follows the inequality (k − 1)l ≤ n − l, which is equivalent to the estimate (3). The theorem is proved.

1. If l = 0, then it follows from (3) that k can take any value 1, 2, . . . , which is confirmed by the example Q = 0.

2. For l = n, the Pontryagin-Mishchenko condition is fulfilled if and only if Q is a body, which is equivalent to k = 1.

3. Let 0 < l < n, 1 ≤ k ≤ £ n l ¤. Let's represent the number n as n = lk + r, r ≥ 0 is some integer. Decompose the space R n into a direct sum of k instances of an l-dimensional and one r-dimensional space:

R l 1 ⊕ · · · ⊕ R l k ⊕ R r .

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At this point, we simplify the condition of rotatability. First of all, we note that for fixed R n, L, C, the set Q satisfies or does not satisfy the condition of rotatability simultaneously with the set Q + a (a is an arbitrary vector). Therefore, we can assume that 0 ∈ Q.

Let for any two-dimensional subspace W the set T is not a body in W. Fixing W. Due to the two-dimensionality of the subspace W, the set T lies on some line W1 ⊂ W.By convention 0 ∈ Q means 0 ∈ T, so W is a one-dimensional subspace. Let v be an arbitrary vector Q. We have

πetC v = πv + τπCv + · · · + τ m m! πCmv + · · · (8)

By the Hamilton-Cayley theorem, the map C m for m ≥ n is a linear combination of maps E, C, . . . , Cn−1, so in decomposition (8) each term is linearly expressed in terms of πv, πCv, . . . , πCn−1 v.But it follows from the assumption T ⊂ W1 that πCiQ ⊂ W1 for i ≤ n − 1,therefore, for all τ ≥ 0, πetC Q ⊂ W1 takes place. Thus, any two-dimensional subspace W contains a one-dimensional subspace W1 is such that for all τ ≥ 0πetC Q ⊂ W1, which contradicts the condition of rotatability.

Let the rotatability condition not be fulfilled. Any two-dimensional subspace W contains a one-dimensional subspace W1 such that there is an arbitrarily close to zero number τ0 > 0, for which

πetC Q ∗ ⊂ W1, τ ∈ (0, τ0]. (9)

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